The clarity we built tonight
If days 2-7 fell on shabbos can't shake Lulav for Rabbis enacted a gezera that we worrried you'll carry it in the public domain to an expert.
If day 1 fell on Shabbos, Rabbis didn't enforce this gezera above due to special nature of first day.
But
If you in chutz l'aretz since we not sure which day is actually yom tov and thus which day is actually one's obligation for lulav, Rabbis forbade pushing off Shabbos for a 'safek' fulfillment of Lulav.
If you in Israel however you know exactly when yom tov is and when your lulav obligation is, so in fact you can shake lulav on Shabbos on first day yom tov in Israel.
BUT
The Rabbis made a lo plug and standardised the Halacha for entire Jewish world. Hence they followed chutz l'aretz and even in Israel don't shake on Shabbos on first day yom tov.

Sukkah daf 42b